ACOUSTIC OSCILLATIONS IN A CAVITY WITH SOURCES AND SINKS

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This paper describes a method of obtaining the relaxation time of free acoustic oscillations in a cavity of arbitrary geometry and also the stability limit directly from a consideration of the boundary problem. The case where the sources and sinks are similar to a solid wall was investigated in [1]. Here we investigate the more general case. The sources and sinks are assumed to be of two types – similar to walls and to holes.

In [2] a similar examination was made from energy considerations. In the final result, however, there was an error – the acoustic energy carried by the main flow was not assessed correctly (see below). A similar error arises out of the general faults of the energy method, which requires a prior knowledge of all the components of the energy flux.

We also consider induced oscillations.

1. Free Oscillations. We consider the following mathematical model of a gas apparatus. We have a three-dimensional region (Ω), filled with gas, but not necessarily simply connected. On the surface of the region there are distributed gas sources. (Henceforth we will use the term source for both sources and sinks; a sink will be regarded as a negative source). The gas in the cavity is in motion. We assume that there is a steady-state regime, i.e., the sum of the outputs of the sources is zero.

The equation for acoustic oscillations in a gas moving with velocities much less than the velocity of sound has the form [3]

$$\Delta\Phi-\frac{2}{c^2}\mathbf{v}\cdot\nabla\Phi-\frac{1}{c^2}\frac{\partial^2\Phi}{dt^2}=0. \qquad (1.1)$$

The acoustic velocity potential Φ is defined so that

$$u=\nabla\Phi, \qquad p=-\rho\frac{\partial\Phi}{\partial t}-\rho\mathbf{v}\cdot\nabla\Phi. \qquad (1.2)$$

Here $v=v(r)$ is the velocity of the main flow, c is the velocity of sound, u and p are the velocity and pressure in the acoustic wave, and ρ is the density of gas filling the cavity.

The boundary condition on the surface (S) is expressed in the form of a linear relationship between the acoustic component of the normal velocity of the gas flow and the acoustic pressure,

$$u_n=B(S)\,p. \qquad (1.3)$$

Here n is the external normal to the surface; B(S) is a quantity characterizing the acoustic properties of the source and is usually called the acoustic conductivity of the surface. Using (1.2) we rewrite condition (1.3) in the form

$$\frac{\partial\Phi}{\partial n}=-\left(\rho\frac{\partial\Phi}{\partial t}+\rho\mathbf{v}\cdot\nabla\Phi\right)B(S). \qquad (1.4)$$

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We consider now the problem of the oscillations given by Eq. (1.1) and (1.4). Assuming $\Phi = Ue^{i\omega t}$ we obtain the problem for determination of the amplitude of the acoustic potential U(r)

$$\Delta U - \frac{2i\omega}{c^2}\mathbf{v}\cdot\nabla U + \frac{\omega^2}{c^2}U = 0 \quad (\Omega) \tag{1.5}$$

$$\partial U/\partial n = -(i\omega\rho U + \rho\mathbf{v}\cdot\nabla U)B \quad (S) \tag{1.6}$$

Note. Condition (1.6) describes not only quasi-steady laws of the type (1.3), but also unsteady laws. For small oscillations the unsteadiness of the law is manifested in a phase shift between u_n and p. In this case $B = B_r + iB_i$ is a complex quantity which depends on the frequency. For the quasi-steady case B is a real number. For a source $B < 0$, for a sink $B > 0$, and for an ideally rigid, nonabsorbing wall $B = 0$.

We will consider sources of two kinds: I) sources with low acoustic conductivity,

$$\rho c B \ll 1 \tag{1.7}$$

i.e., similar to a "solid wall". These sources are placed on part of the surface which we will denote by (C); II) sources with a low impedance,

$$\rho c B \gg 1 \tag{1.8}$$

i.e., similar to holes and placed on part of the surface denoted by (O).

Note. A solid wall can be regarded as a type I source for which $\nu_n = 0$. Thus the whole surface of the cavity consists of two parts (S=C+O).

We will also assume that the gas in the cavity moves at low velocities, so that

$$v / c \ll 1 \tag{1.9}$$

Assuming that inequalities (1.7)-(1.9) are fulfilled, we can obviously seek the solution of problem (1.5), (1.6) in the form of a sum

$$U = U_0 + U_1, \ \omega = \omega_0 + \omega_1 \tag{1.10}$$

We assume that

$$U_1 \ll U_0, \qquad \omega_1 \ll \omega_0 \tag{1.11}$$

Here U_0, ω_0 is the solution of the problem in the zero approximation,

$$\Delta U_0 + \frac{\omega_0^2}{c^2}U_0 = 0 \quad (\Omega), \qquad \frac{\partial U_0}{\partial n} = 0 \quad (C), \qquad U_0 = 0 \quad (O) \tag{1.12}$$

We substitute (1.10) in (1.5), (1.6). Neglecting terms of the second order of smallness containing squares and products of U_1, ω_1, and ν in the region (Ω) and on the surface (C) and also the square terms containing U_1/B on the surface (O) we obtain the problem for the determination of U_1 and ω_1:

$$\Delta U_1 + \frac{\omega_0^2}{c^2}U_1 = \frac{2i\omega}{c^2}\mathbf{v}\cdot\nabla U_0 - \frac{2\omega_0\omega_1}{c^2}U_0 \quad (\Omega) \tag{1.13}$$

$$\frac{\partial U_1}{\partial n} = -i\omega_0\rho B U_0 \quad (C) \tag{1.14}$$

$$\frac{1}{B}\frac{\partial U_0}{\partial n} = -i\omega_0\rho U_1 - \rho\mathbf{v}\cdot\nabla U_0 \quad (O) \tag{1.15}$$

Henceforth, we will be concerned only with the determination of ω_1 – the increment to the purely acoustic frequency ω_0. From the sign of the imaginary part of ω_1 we can determine the stability of the system, and from the magnitude of ω_1 we can determine the relaxation time.

We can obtain ω_1 from the problem (1.13)-(1.15) by the following method. We consider Green's integral identity for the region (Ω), applied to functions U_0 and U_1.

$$\iiint_{\Omega} (U_0 \Delta U_1 - U_1 \Delta U_0)\, d\Omega = \iint_{C+O} \left(U_0 \frac{\partial U_1}{\partial n} - U_1 \frac{\partial U_0}{\partial n}\right) dS \tag{1.16}$$

We substitute in (1.16) the expression for ΔU_1 from (1.13), the expression for ΔU_0 from (1.12), the value of $\partial U_1/\partial n$ on (W) from (1.14), and U_1 on (H) from (1.15). We also take into account that on the surface (W) $\partial U_0/\partial n=0$ and on the surface (H) $U_0=0$. Then from (1.16) we obtain the expression for the increment ω_1:

$$\omega_1 = \frac{i}{J}\left\{2\iiint_{\Omega} U_0 (\mathbf{v}\cdot\nabla U_0)\, d\Omega + \iint_{C} \rho c^2 B U_0^2\, dS\right\} + $$
$$+ \frac{i}{J}\frac{c^2}{\omega_0^2}\left\{\iint_{O}\left[\frac{1}{\rho B}\left(\frac{\partial U_0}{\partial n}\right)^2 + (\mathbf{v}\cdot\nabla U_0)\frac{\partial U_0}{\partial n}\right] dS\right\}$$
$$J = 2\iiint_{\Omega} U_0^2\, d\Omega \tag{1.17}$$

We convert (1.17) to a more convenient form. We consider the identity

$$\operatorname{div}(\nu U_0^2) = U_0^2 \operatorname{div}\mathbf{v} + 2(\mathbf{v}\cdot\nabla U_0)\, U_0 \tag{1.18}$$

Assuming that there are no sources in the volume and the gas can be regarded as incompressible for $\nu/c \ll 1$, we conclude that $\operatorname{div}\mathbf{v}=0$. Then, by using the Gauss divergence formula and (1.18) we obtain

$$2\iiint_{\Omega} U_0 (\mathbf{v}\cdot\nabla U_0)\, d\Omega = \iint_{C+O} v_n U_0^2\, dS \tag{1.19}$$

Next, from the fact that on the surface (H) the function $U_0 \equiv 0$, it follows that $\partial U_0/\partial e_{1,2}=0$ (here $e_{1,2}$ are unit vectors tangential to the surface). Hence,

$$\nabla U_0 = \frac{\partial U_0}{\partial n}\mathbf{n} + \frac{\partial U_0}{\partial e_1}\mathbf{e}_1 + \frac{\partial U_0}{\partial e_2}\mathbf{e}_2 = \frac{\partial U_0}{\partial n}\mathbf{n} \tag{1.20}$$

Using (1.19) and (1.20) we obtain from (1.17) the main expression for investigating the stability of the system:

$$\omega_1 = \frac{ic}{J}\iint_{C}\left(\rho c B + \frac{v_n}{c}\right) U_0^2\, dS + \frac{i}{J}\frac{c^3}{\omega_0^2}\iint_{O}\left(\frac{1}{\rho c B} + \frac{v_n}{c}\right)\left(\frac{\partial U_0}{\partial n}\right)^2 dS \tag{1.21}$$

Here we deliberately avoid cancelling the c so that we separate the dimensionless quantities ρcB and ν_n/c.

The literature contains special cases of formula (1.21) for the simplest one-dimensional problem and for the cylindrical problem in the absence of a flow.

It follows from (1.21) that to investigate the stability limit we need to know: 1) the acoustic properties of the sources (B); 2) the normal component of the velocity of the main flow at the boundary of the region (ν_n); 3) the acoustic field in the absence of sources and sinks (U_0).

If $\mathrm{Im}\,\omega_1 > 0$, the process is stable, and if $\mathrm{Im}\,\omega_1 < 0$ it is unstable. Assuming $\mathrm{Im}\,\omega_1=0$ we can find the stability limit of the system.

We will discuss more fully the physical sense of Eq. (1.21). The first term represents the interaction of sound waves with sources of the wall type. The denominator contains the expression J, which is proportional to the total energy of acoustic oscillations in the volume. The numerator is proportional to the flux of acoustic energy through the surface C in a period. This flux consists of two components. The first gives the energy flux carried by the waves themselves and the second that carried by the main flow. That energy can also enter and leave a cavity when sources interact with an acoustic field is a well-known fact. It is accepted also that the main flow leaving the cavity carries off acoustic energy. Expression (1.21), however, embodies a fact which was not obvious beforehand – that acoustic energy can be generated by the main flow when it enters the antinode of the pressure wave. (Since in the sources $\nu_n < 0$ the contribution of the corresponding terms in $\mathrm{Im}\,\omega_1$ is negative.) Rayleigh [4] was the first to point out the general possibility of such generation.

Raushenbakh [5] showed from energy considerations that such generation was possible in the one-dimensional case. We point out that although generation by the flow is expressed by a surface integral it is a volume factor which is not connected with boundary conditions, but with the convective term in the wave equation. We note also that in some special cases the flow has no effect on the stability of the system as a whole. This occurs when

$$\iint\limits_C v_n U_0^2\, dS = 0, \qquad \iint\limits_O v_n \left(\frac{\partial U_0}{\partial n}\right)^2 dS = 0$$

The second term in (1.21) represents the interaction of the field with sources of the hole type. The first term in the numerator gives the energy flux carried across the surface (O) by waves, and the second term gives that carried by the main flow. As already mentioned, the authors of [2] omitted the term $\nu_n/c(\partial U_0/\partial n)^2$, which represents sound generation by the flow entering an antinode of the velocity wave and the removal of energy by the flow leaving this antinode. We note that the generation represented by this term depends on the boundary condition (1.15), i.e., is a surface effect.

By the proposed formalism we can easily obtain corrections to the Eq. (1.21) for the various additional factors taken into account in the theory of the process, if they have little effect on the acoustics, which is what usually occurs in practice. We can easily take into account the nonuniformity of the sound velocity, absorption in the volume, volume sources, etc. For instance, if we take absorption into account by means of a complex sound velocity $c = c_0 + i\sigma$ $(\sigma \ll c_0)$, then in the final result (1.21) we will have a correction,

$$\frac{i}{J}\frac{\omega_0}{c}\iiint\limits_\Omega \sigma U_0^2\, d\Omega \tag{1.22}$$

It is clear that absorption always stabilizes the process (the contribution to $\mathrm{Im}\,\omega_1$ is positive). Formula (1.21) automatically takes into account the case where the acoustic conductivity of the surface depends on the angle of incidence of the wave (ϑ). For this we merely formally replace B by $B(\vartheta)$.

We note that for correctness of the results of the proposed formalism, as A. D. Margolin showed, the local inequalities (1.7)-(1.9) are, generally speaking, not sufficient. The integral inequalities obtained from Eq. (1.21) will have to be fulfilled. In fact, it was assumed in the deduction that $\omega_1 \ll \omega_0$. This is possible if [see (1.21)]

$$\frac{c}{J\omega_0}\iint\limits_C \left(\frac{v_n}{c} + \rho c B\right) U_0^2\, dS \ll 1 \tag{1.23}$$

$$\frac{c^3}{J\omega_0^2}\iint\limits_O \left(\frac{v_n}{c} + \frac{c}{\rho c B}\right)\left(\frac{\partial U_0^2}{\partial n}\right) dS \ll 1 \tag{1.24}$$

For instance, for longitudinal oscillations in a cylindrical cavity in which the sources are situated on the side surface, and a sink on the ends, we obtain from (1.23) the inequality

$$\left(\rho c B + \frac{v_n}{c}\right)\frac{l}{d} \ll 1$$

Here l is the length of the cavity and d is its diameter. Thus, the present theory cannot be applied in the case of a very long cylinder. This corresponds physically to the case where the energy accumulated in the middle of the cavity during a period cannot be transferred to the walls during this period.

2. Induced Oscillations. We will assume that oscillations are excited on the boundary of the cavity by a periodic change in the velocity of the gas outflow. Such a case occurs in the case of an oscillating membrane or siren. (The case where the pressure changes on the boundary of the region can be considered in the same way.)

Instead of (1.3) we will have the boundary condition

$$\frac{\partial \Phi}{\partial n} = -\left(\rho \frac{\partial \Phi}{\partial t} + \rho \mathbf{v} \cdot \nabla \Phi\right) B(S) - A(S) e^{i\omega t} \tag{2.1}$$

where A(S) – the amplitude of the induced velocity oscillations – is a fairly small quantity, characterized by the inequality

$$A / c \ll 1 \tag{2.2}$$

We write the solution in the form of a steady process $\Phi = Ue^{i\omega t}$. The problem of finding the amplitude U then reduces to solution of Eq. (1.5) with conditions

$$\frac{\partial U}{\partial n} = -i\omega\rho BU - A \qquad (C) \tag{2.3}$$

$$\frac{1}{B}\frac{\partial U}{\partial n} = -i\omega\rho U - \rho \mathbf{v} \cdot \nabla U \qquad (O) \tag{2.4}$$

In the deduction of (2.3) and (2.4) we neglect terms of the second order of smallness, as was done in the deduction of (1.14), (1.15). We note that in (2.4) the term containing the excitation amplitude A has disappeared. This means that generation is produced only by sources of the wall type. (In the case of excitation by pressure oscillations, on the other hand, sources of the hole type are important.)

We apply Green's identity to functions U and G, where G is an arbitrary function at present:

$$\iiint_{\Omega} (U\Delta G - G\Delta U)\, d\Omega = \iint_{S} \left(U \frac{\partial G}{\partial n} - G \frac{\partial U}{\partial n}\right) dS$$

Substituting here ΔU from (1.5), $\partial U/\partial n$ from (2.3), and U from (2.4), we have

$$\iiint_{\Omega} U\left(\Delta G + \frac{\omega^2}{c^2} G\right) d\,\Omega - \iiint_{\Omega} G\, \frac{2i\omega}{c^2} \mathbf{v} \cdot \nabla U\, d\Omega =$$

$$= \iint_{C} \left[\frac{\partial G}{\partial n} + G(i\omega BU + A)\right] dS + \iint_{O} \left[\frac{\partial G}{\partial n}\left(\frac{1}{B}\frac{\partial U}{\partial n} + \rho \mathbf{v} \cdot \nabla U\right)\frac{i}{\omega p} - G\frac{\partial U}{\partial n}\right] dS \tag{2.5}$$

It follows from (2.5) that the solution of the problem for U depends considerably on whether ω is close to the frequency of natural oscillations in the purely acoustic problem (1.12).

Case 1. The quantity ω is not the natural frequency. It can be seen from (2.5) that U is of the same order of smallness as A. Neglecting terms of the second order of smallness, we obtain

$$\iiint_{\Omega} U\left(\Delta G + \frac{\omega^2}{c^2} G\right) d\Omega = \iint_{C} U \frac{\partial G}{\partial n} dS - \iint_{O} G \frac{\partial U}{\partial n} dS + \iint_{C} GA\, dS \tag{2.6}$$

Since G is an arbitrary function, U is the so-called generalized solution [6] of the problem

$$\Delta U + \frac{\omega_0^2}{c^2} U = 0 \quad (\Omega), \qquad \frac{\partial U}{\partial n} = -A \quad (C), \qquad U = 0 \quad (O) \tag{2.7}$$

It follows from (2.6) or (2.7) that the oscillations occur as if there were no sources or sinks. The amplitude of the oscillations is a small quantity of the order of A.

Case 2. The quantity ω is often close to the natural frequency ω_0 of the problem (1.12), i.e., $\omega=\omega_0+\omega_1$ ($\omega_1 \ll \omega_0$). In this case we can take $G \equiv U_0$, where U_0 is the solution of problem (1.12). In this case (2.5) is converted to

$$\frac{2\omega_0\omega_1}{c^2}\iiint\limits_{\Omega} U U_0\, d\Omega - \frac{2i\omega_0}{c^2}\iiint\limits_{\Omega} U_0 (\mathbf{v}\cdot\nabla U)\, d\Omega = \\ = \iint\limits_{C} U_0 (i\omega_0\rho B U + A)\, dS + \iint\limits_{O} \frac{\partial U}{\partial n}\left(\frac{1}{B}\frac{\partial U}{\partial n} + \rho\mathbf{v}\cdot\nabla U\right)\frac{i}{\omega_0\rho}\, dS \qquad (2.8)$$

The latter does not determine U uniquely and, hence, we use the fact that

$$U(\mathbf{r}) = U_0(\mathbf{r})\,\text{const} \qquad (2.9)$$

which is accurate to quantities of the first order of smallness in comparison with U_0. This follows from a consideration of the problems (1.5), (2.3), and (2.4) by the Fourier method and means physically that the form of the induced oscillations at resonance frequency is the same as the corresponding form of the free oscillations. Substituting (2.9) in (2.8) we find the constant and, hence, U(r). Using (1.19) and (1.20), we write the final expression for the amplitude of induced oscillations at frequency $\omega_0+\omega_1$:

$$U(r) = i\frac{c}{\omega_0}\iint\limits_{C} A U_0\, dS \cdot U_0(r)\left[\iint\limits_{C}\left(\rho c B + \frac{v_n}{c}\right)U_0^2\, dS + \right. \\ \left. + \frac{c^2}{\omega_0^2}\iint\limits_{O}\left(\frac{1}{\rho c B} + \frac{v_n}{c}\right)\left(\frac{\partial U_0}{\partial n}\right)^2 dS + 2i\frac{\omega_1}{c}\iiint\limits_{\Omega} U_0^2\, d\Omega\right]^{-1} \qquad (2.10)$$

The amplitude of such oscillations is an order higher than in case 1 and conveys information about the sources. We note that the formula is true only for a stable system. Near the stability limit the denominator tends to zero, and the solution becomes nonsense if terms of the second order of smallness are neglected.

For practical purposes it is convenient to have (2.10) in real notation. For the pressure amplitude we have the expression $P=-i\omega\rho U$ (here we omit the small term $\sim \mathbf{v}\cdot\Delta U$). We also take into account absorption in the volume, as was done in the deduction of (1.22). If the rate of induced vibrations of the velocity varies according to a $\sim\cos\omega t$ law, the pressure varies as $\sim\cos(\omega t-\Psi)$.

The pressure amplitude P and phase shift Ψ are, respectively,

$$P = \frac{\rho c^2}{\sqrt{D^2+E^2}}\iint\limits_{C} A P_0\, dS \cdot P_0(r), \qquad \psi = -\operatorname{arc\,tg}\frac{E}{D} \qquad (2.11)$$

$$D = c\iint\limits_{C}\left(\rho c B_r + \frac{v_n}{c}\right)P_0^2\, dS + \frac{c^3}{\omega_0^2}\iint\limits_{O}\left[\frac{1}{\rho c}\left(\frac{1}{B}\right)_r + \frac{v_n}{c}\right]\left(\frac{\partial P_0}{\partial n}\right)^2 dS + \\ + 2\omega_0\iiint\limits_{\Omega}\frac{\sigma}{c}P_0^2\, d\Omega$$

$$E = 2\omega_1\iiint\limits_{\Omega} P_0^2\, d\Omega - c\iint\limits_{C}\rho c B_i P_0^2\, dS - \frac{c^3}{\omega_0^2}\iint\limits_{O}\frac{1}{\rho c}\left(\frac{1}{B}\right)_i\left(\frac{\partial P_0}{\partial n}\right)^2 dS \qquad (2.12)$$

Here,

$$B_r = \operatorname{Re} B, \quad \left(\frac{1}{B}\right)_r = \operatorname{Re}\left(\frac{1}{B}\right), \quad B_i = \operatorname{Jm} B, \left(\frac{1}{B}\right)_i = \operatorname{Jm}\left(\frac{1}{B}\right)$$

We note that P_0 differs from the solution U_0 of problem (1.12) only by an insignificant constant factor.

It follows from (2.11) that in the case of a slow-moving (quasi-steady) position of the resonance the maximum is attained at a frequency of $\omega_0+\omega_2$, where

$$\omega_2 = \frac{1}{2}\left[c\iint\limits_C \rho c B_i P_0^2\,dS + \frac{c^3}{\omega_0^2}\iint\limits_O \frac{1}{\rho c}\left(\frac{1}{B}\right)_i\left(\frac{\partial P_0}{\partial n}\right)^2 dS\right]\left(\iiint\limits_\Omega P_0^2\,d\Omega\right)^{-1} \quad (2.13)$$

Equations (2.11)-(2.13) can be used to determine the acoustic characteristics $B_r(\omega)$ and $B_i(\omega)$ of the sources. For an investigation of the stability it is important to know the real part of the acoustic conductivity on a surface of the wall type and the real part of the impedance on a surface of the hole type. The most convenient experimental method of determining these quantities consists in the following. The frequency ω of the induced oscillations is varied slowly in a quasi-steady manner, so that the system passes through resonance. The amplitude of the induced oscillations passes through a maximum, which is symmetric relative to the frequency $\omega_0+\omega_2$. It follows from (2.11)-(2.13) that the real part of the acoustic characteristics can be determined uniquely, for instance, from the width of the resonance peak $(2\omega^*)$ at half height,

$$2\omega^* = \sqrt{3}\left\{c\iint\limits_C\left(\rho c B_r + \frac{v_n}{c}\right)P_0^2\,dS + \frac{c^3}{\omega_0^2}\iint\limits_O\left[\frac{1}{\rho c}\left(\frac{1}{B}\right)_r + \frac{v_n}{c}\right]\left(\frac{\partial P_0}{\partial n}\right)^2 dS + \right.$$
$$\left. + 2\omega_0\iiint\limits_\Omega \frac{\sigma}{c}P_0^2\,dS\right\}\left(\iiint\limits_\Omega P_0^2\,d\Omega\right)^{-1} \quad (2.14)$$

The imaginary parts of the acoustic characteristics of the sources do not affect the shape of the peak and merely lead to a shift in relation to frequency. For the one-dimensional case this method of determining the acoustic characteristics is well known in acoustics [7].

Equation (2.14) gives an integral characterization of the sources. Hence, to determine the characteristics of one of the sources we need to know in the general case the characteristics of the other sources. In special cases, where sources of the wall type are placed at the pressure nodes, and sources of the hole type are placed at the velocity nodes, the characteristics of these sources need not be known.

LITERATURE CITED

1. Yu. I. Babenko and O. M. Todes, "Stability of acoustic oscillations in cavities with sources and sinks," Kotloturbostroenie. Tr. Tsentr. Kotloturbinnogo in-ta, no. 64, 1965; "Acoustic oscillations in chemical gas reactors," collection: Research on Thermodynamics and the Kinetics of Chemical Processes [in Russian], Khimiya, Moscow-Leningrad, no. 54, 1966.
2. R. W. Hart and F. T. McClure, "Theory of acoustic instability in solid-propellant rocket combustion," Preprint. 10-th Sympos. (Juteruat) on Combustion, Comb. inst., Pittsburgh, Pennsylvania, 3a [N 1], 15/1-15/18, 1965.
3. D. I. Blokhintsev, Acoustics of a Non-Uniform Moving Medium [in Russian], Gostekhizdat, Moscow-Leningrad, 1946.
4. J. W. S. Rayleigh, Theory of Sound [Russian translation], vol. 2, Gostekhizdat, Moscow, 1955.
5. B. V. Raushenbakh, Vibrational Combustion [in Russian], Fizmatgiz, Moscow, 1961.
6. O. A. Ladyzhenskaya, Mathematical Aspects of the Dynamics of a Viscous Incompressible Liquid [in Russian], Fizmatgiz, Moscow, 1961.
7. E. Skudrzyk, Grundlagen der Akustik, [Russian translation], vol. 2, Izd-vo inostr. lit., Moscow, 1959.